Estimation of initial velocity, angle of launch, and range of a shuttlecock for a given badminton stroke

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1. Objectives

- To estimate initial velocity, angle of launch and range of a shuttlecock for a given badminton stroke, so that a player can achieve the best outcome from a stroke against an opponent.

- To apply the flight trajectory equation developed by Chen et al and understand the effect of the air drag (resistance force) on the shuttle as validated by Alam.F et al.

2. Introduction

Badminton is the fastest racquet sport and is popular world-wide. Player movement analysis and prediction of shuttle dynamics such as smashing, service, racket swing and shuttlecock trajectory has seen significant improvement with accuracy over the past few years with the betterment of technology.

What makes the flight of a shuttlecock unique, and it's influence on the game is an intriguing topic of research for the sport-lovers, experts and scientists. The properties (rotation, tumbling, turn-over, trajectory) and design of shuttlecocks are still an active field of research.

The shuttlecock is known to generate significant aerodynamic drag and has a complex flight trajectory which is approximately a skewed parabola.

The cone comprises of 16 overlapping goose feathers embedded into a round cork base, enclosed with a thin leather or synthetic material. Most amateur players use synthetic shuttlecock as it lasts longer and exhibits less aerodynamic drag compared to feather shuttlecock which is predominantly used by the professional players and have high initial velocity.

Fig 1.a: Feather shuttlecock

Fig 1.b: Synthetic shuttle cock

Image Source: Wikipedia, shuttle badminton

3. Relevant Information

3.1 Drag coefficient variation of a shuttlecock

From the studies validated by Alam.F et al, a feather shuttle is found to have lower drag coefficient value at low speeds and a higher value at high speeds. On the other hand, a synthetic shuttle is shown to have an opposite trend.

The experiments were carried out in a wind tunnel and the trajectory and speeds were captured using a high-resolution motion camera. Standard Yonex feather shuttle as prescribed by the Badminton World Federation (BWF), with weight ~ 5g, length of shuttle ~85mm, length of the cock ~ 25mm, width at the end of the skirt ~ 65mm were used.

The mass is concentrated around the cork, such that the centre of gravity (C.G.) is just behind the base of the cork. The thin-walled conical skirt of the badminton shuttlecock generates large aerodynamic drag force that acts through the centre of pressure (C.P.). The combination of a forward C.G. and a rearward C.P. gives the badminton shuttlecock a natural tendency to fly nose first in a stable manner.

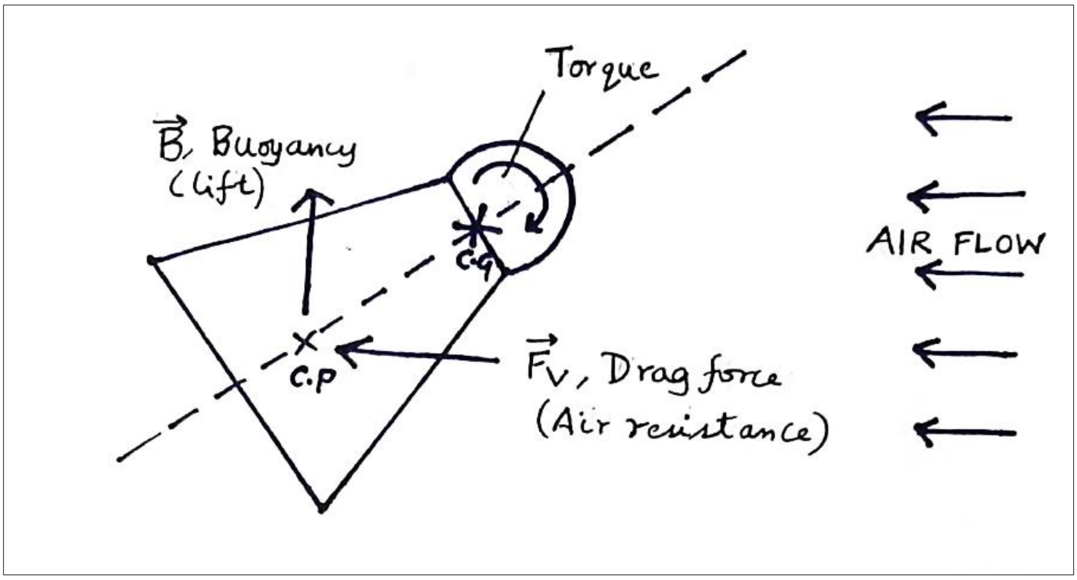

Fig 2: Forces on a shuttlecock

The aerodynamic drag force was found to be-

$F_V = \frac{1}{2} \times (C_D \rho v^2 A)$

Where, C_D is the drag coefficient, ρ is the air density, v is the wind speed, and A is the undeformed projected frontal area of a shuttlecock.

The Reynolds number (R_E) for a shuttlecock was defined as-

$$R_E = \frac{(\upsilon d)}{\nu}$$

Where, υ is the wind speed, d is the skirt diameter, ν is the kinematic viscosity.

Shuttlecocks were tested at 40 to 130 km/h speeds with an increment of 10 km/h and the drag coefficient variation (C_D) was plotted vs Reynolds number (R_E).

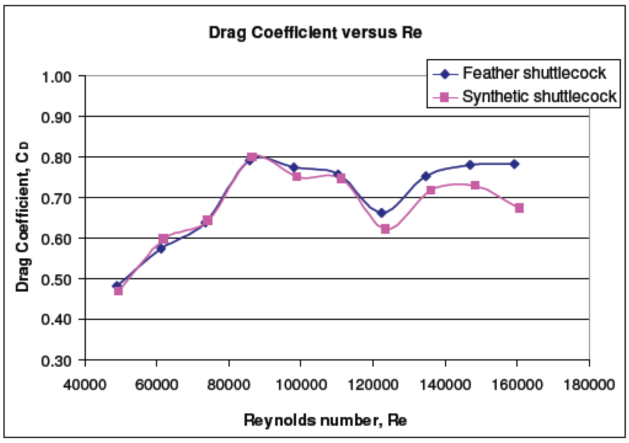

Fig 3: C_D vs R_E for feather and synthetic shuttlecock.

Image source: Alam, F et al

The average C_D value for all shuttlecocks is lower at low Reynolds number initially and increases with an increase of R_E. However, the C_D value drops over 80km/h.

The average drag coefficient for all shuttlecocks tested is approximately 0.61 over 100 km/h and 0.51 at 60 km/h.

There is significant variation in drag coefficients among the synthetic shuttlecocks which is due to varied geometry of skirts and deformation at high speeds. On the other hand, less variation of drag coefficients was observed for feather shuttlecocks. The variation in C_D is minimal for the feather shuttlecock due to less deformation at high speeds and also less variation in skirt geometry. The average C_D value for feather shuttlecock is higher at low speeds compared to synthetic shuttlecocks. In contrast, the average C_D value for the synthetic shuttlecock is higher at high speeds compared to the C_D value for feather shuttlecock.

For commercial shuttlecocks, C_D varies between 0.6 and 0.7 depending on the material and design of the skirt.

3.2 Badminton court dimensions

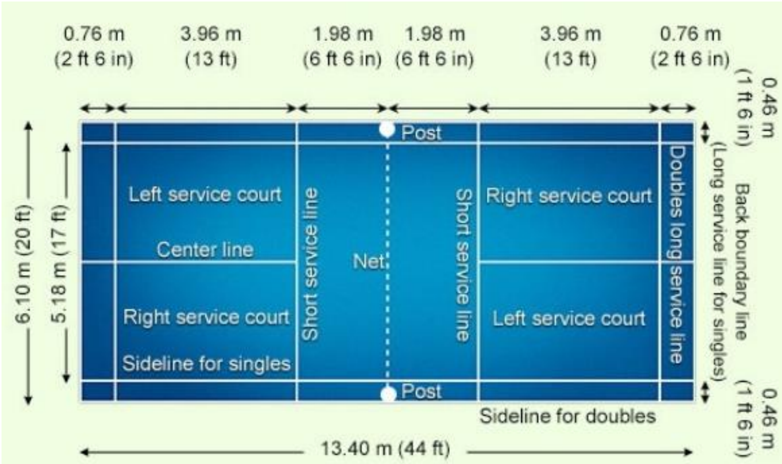

Fig 4: Badminton court dimensions

Image source: https://dailybadminton.com/badminton-court-dimension/

The badminton court is rectangular and laid out with line markings 40mm wide, preferably in white or yellow colour. The court is 13.4m in length and 6.1m in width. The post is approximately 1.55m in height from the surface of the court and remains vertical when the net is strained.

3.3 Badminton stroke types

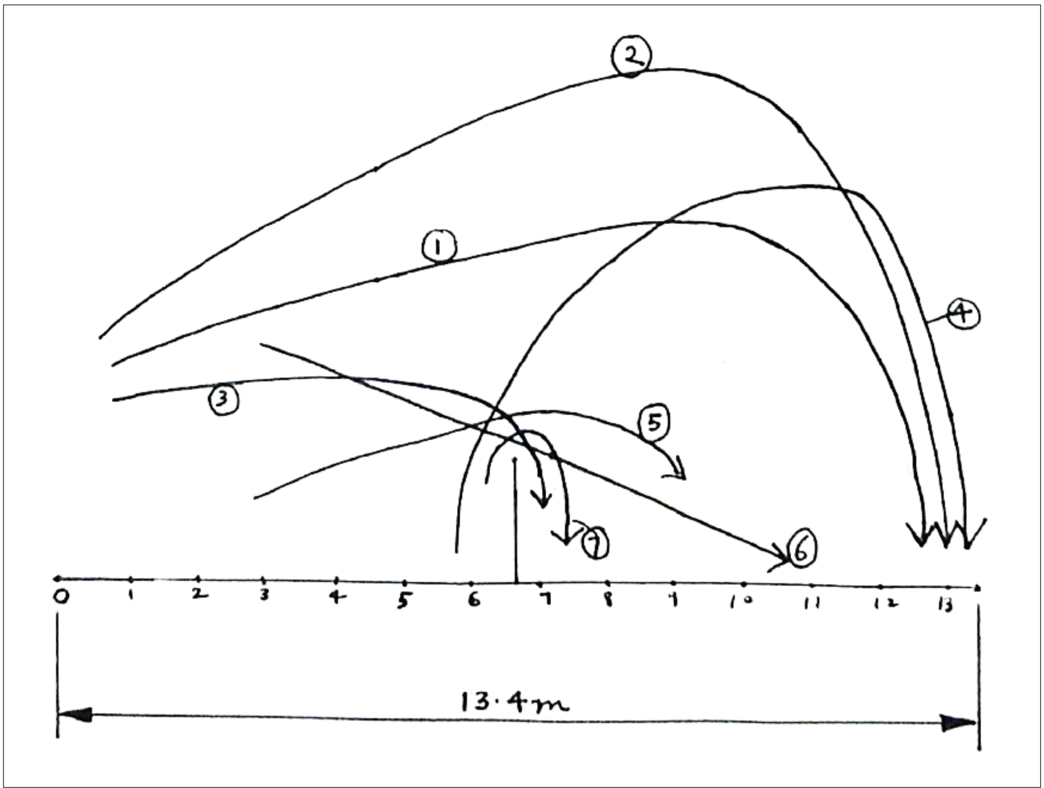

Fig 5: Side view of the court showing the different shuttlecock trajectories during a game.

A clear toss (1) and (2) can be an offensive stroke (1), moving the opponent back from the net or a defensive one (2), saving time to improve the player's position.
Drops (3) and net shots (7) are slow, gentle shots that fall just behind the net into the opponent's court.
A lift (4) is actually an underarm clear played from around the net area. This shot allows one to move the opponent to the back or to save time.
The drive (5) is a line-drive shot parallel to the ground passing just over the net.
The smash (6) is a fast ball with a sharp straight trajectory aimed either at the opponent's body or at the limits of the court.

4. Equation for trajectory of shuttlecock

Chen et al (2009) constructed a motion equation of a shuttlecock's flying trajectory under the effects of gravitational force and air resistance force ignoring spin effects. The result showed that the motion equation of a shuttlecock's flight trajectory could be constructed by determining the terminal velocity. The terminal velocity of the shuttlecocks ranged from 6.51 to 6.87 m/s.

The predicted shuttlecock trajectory could fit the measured data fairly well. The results also revealed that the drag force was proportional to the square of a shuttlecock velocity. They also proposed that the quadratic drag force was influenced by the Reynolds number R_E.

4.1 Forces on the shuttlecock

From Newton's II Law, when a shuttlecock is in flight-

$\vec{W} + \overrightarrow{Fv} + \vec{B} = m\,\vec{a}$

Where, $\vec{W}$ is the gravitational force, $\overrightarrow{Fv}$ is the aerodynamic drag (air resistance force), $\vec{B}$ is the buoyancy, m is the mass of the shuttlecock and $\vec{a}$ is the acceleration of the shuttlecock.

Assumption: The magnitude of $\vec{B}$ is very small in comparison with $\vec{W}$ and $\overrightarrow{Fv}$, and hence it's influence can be neglected. Therefore,

$\vec{W} + \overrightarrow{Fv} = m\,\vec{a}$ ………………………………………………………. (1)

$\overrightarrow{Fv}$ depends on the relative speed of the shuttlecock with respect to air. The direction of $\overrightarrow{Fv}$ is always opposite to the direction of motion of the shuttle.

According to Thorton & Marion (2003), the drag force can be expressed as

$F_v = bv^n$ …………………………………………………………………. (2)

Where, v is the speed of the shuttlecock relative to air, n and b are real constants that depend on the properties of air and the shape and dimension of the shuttlecock.
From the experimental data, it is estimated that the quadratic drag force i.e. n=2 best fits the data. Therefore,
$F_v = bv^2$

From (2), it is evident that as the velocity of the shuttle increases, the drag force will also increase.

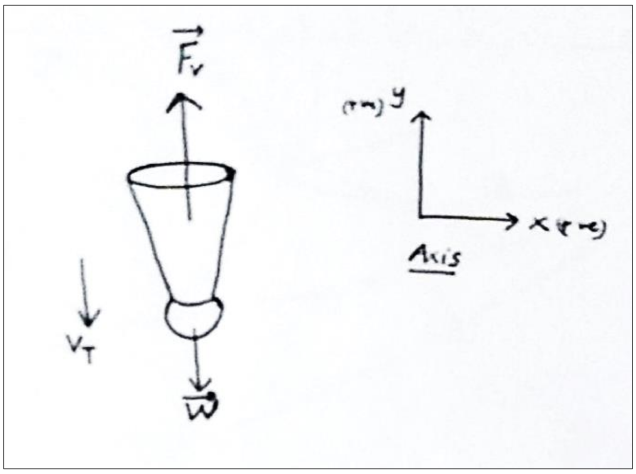

Fig 6: Shuttle reaching terminal velocity

The acceleration will be zero when the drag (air resistance) balances the weight. At this point, the shuttle will be falling down vertically with zero acceleration and would have reached terminal velocity (v_T).

Terminal velocity (v_T) can be measured from (1) and setting $\vec{a} = \frac{dv}{dt} = 0$

$mg - bv_T = 0$

$v_T = \sqrt{\frac{mg}{b}}$ .... (3)

Therefore, measuring v_T of the shuttle can be used to find the parameter 'b'.

4.2 Formulating the equation

Let the shuttlecock be hit with initial velocity v_i and initial angle θ_i (w.r.t horizontal), then horizontal and vertical velocity can be expressed as-

$v_{xi} = v_i \cos\theta_i$

$v_{yi} = v_i \sin\theta_i$

Drag force can be expressed as-

$F_v = F_{vx}\,\hat{\imath} + F_{vy}\,\hat{\jmath}$

$F_v = bv^2\cos\theta\,\hat{\imath} + bv^2\sin\theta\,\hat{\jmath}$

Considering the motion in y-direction,

$$F_{v_y} = m\frac{dv_y}{dt} = -mg - bv_y^2 \qquad \text{………………. (4)}$$

Let v_{yi} be the initial velocity at t=0, and v_y be the velocity at time t, and v_T is the terminal velocity as previously described-

$$\int_{v_{yi}}^{v_y} \frac{dv_y}{\left(1 + \frac{bv_y^2}{mg}\right)} = \int_0^t -\,dt \qquad \text{……………. (5)}$$

Solving (5), we obtain

$$\boxed{v_y = \frac{v_{yi} - v_t \tan\left(\frac{gt}{v_t}\right)}{1 + \frac{v_{yi}}{v_t}\tan\left(\frac{gt}{v_t}\right)}} \qquad \text{………… (6)}$$

At the highest point, $v_y = 0$

Therefore, the time of flight (T_f),

$$\boxed{T_f = \frac{V_t}{g}\tan^{-1}\left(\frac{V_{yi}}{V_t}\right)} \qquad \text{…………… (7)}$$

From (6),

$\frac{dy}{dt}$ = v_y

Let the shuttle be released from an initial height y_0 (player height and racquet length depending on stroke)

$$y - y_0 = \int_0^t \frac{\left(v_{yi} - v_t \tan\left(\frac{gt}{v_t}\right)\right)}{\left[1 + \frac{v_{yi}}{v_t}\tan\left(\frac{gt}{v_t}\right)\right]} dt$$

…………… (8)

Solving (8),

$$y = y_0 + \frac{v_t^2}{g} \ln\left[\frac{\sin\left[\frac{gt}{v_t} + \tan^{-1}\left(\frac{v_t}{v_{yi}}\right)\right]}{\sin\left[\tan^{-1}\left(\frac{v_t}{v_{yi}}\right)\right]}\right]$$

……………. (9)

Considering motion in x-direction,

$$F_{vx} = m\frac{dv_x}{dt} = -bv_x^2$$

…………………. (10)

Let v_{xi} be the initial velocity at t=0, and v_x be the velocity at time t, and v_T is the terminal velocity as previously described-

Solving (10),

$$V_x = \frac{V_{xi} V_t^2}{V_{xi} g t + V_t^2}$$

…………………. (11)

From (11),

$\frac{dx}{dt}$ = v_x

$$x = \frac{V_t^2}{g} \ln\left[\frac{V_{xi} g t + V_t^2}{V_t^2}\right]$$

……………………. (12)

Combining (9) and (12) (consider $y_0 = 0$), we obtain the equation of the trajectory,

$$y = \frac{V_t^2}{g} \ln \left[\frac{\sin\left[\frac{V_t}{V_{xi}} \left(e^{\frac{gx}{V_{x}^{2}}} - 1 \right) + \tan^{-1}\left(\frac{V_t}{V_{yi}} \right) \right]}{\sin\left[\tan^{-1}\left(\frac{V_t}{V_{yi}} \right) \right]} \right] \quad \text{………. (13)}$$

5. Optimising initial velocity, angle and range for the best stroke

Tsai, Huang, and Jih (1997) conducted research on elite Taiwanese badminton players and suggested that the initial shuttlecock velocities for the strokes of were-
smashes: 55-70 (m/s), with an average of 62.12 (m/s);
jump smashes: 55-75 (m/s) with an average of 68.16 (m/s);
clears: 42-51 (m/s) with an average of 47.76 (m/s),
drops: 22-29 (m/s) with an average of 25 (m/s).

Consider a clear stroke i.e. a toss.

A toss would be considered as a good stroke if it is hit from the base-line of one side of the court to the base-line of the other end, with sufficient height. Therefore, we should expect horizontal range ~ between 11-13m as the length of the court is 13.4m.

Considering, an average Taiwanese badminton player, who can hit a clear toss at initial speed of 47.76 m/s. If we assume that the shuttle is released from a height of 1.75m above the ground (considering player height and racquet length), and that the player hits at angle of 30^o with respect to the ground. The terminal velocity (v_T) can be considered to be ~6.5m/s. (as obtained from experiments by Chen et al)

$V_i = 47.76\ m/s \qquad \theta_i = 30^\circ$

$V_{xi} = V_i \cos\theta_i = 41.36\ m/s$

$V_{yi} = V_i \sin\theta_i = 23.88\ m/s$

$V_t = 6.5\ m/s \qquad y_0 = 1.75\ m \qquad x_0 = 0$

Time required to reach highest point of trajectory -

From (1) $$T_f = \frac{V_t}{g} \tan^{-1}\left(\frac{V_{yi}}{V_t}\right)$$

$$T_f = \frac{6.5}{9.8} \tan^{-1}\left(\frac{23.88}{6.5}\right) = \frac{6.5}{9.8} \times 1.3$$

$$\underline{T_f = 0.866\ sec}$$

The maximum height reached by the shuttle-cock,

From (9),
$$y - y_0 = \frac{v_t^2}{g} \ln\left[\frac{\sin\left[\frac{gt}{v_t} + \tan^{-1}\left(\frac{v_t}{v_{yi}}\right)\right]}{\sin\left[\tan^{-1}\left(\frac{v_t}{v_{yi}}\right)\right]} \right]$$

$$y - 1.75 = \frac{(6.5)^2}{9.8} \ln\left[\frac{\sin\left[\frac{9.8 \times (0.866)}{6.5} + \tan^{-1}\left(\frac{6.5}{23.88}\right)\right]}{\sin\left[\tan^{-1}\left(\frac{6.5}{23.88}\right)\right]} \right]$$

$$y - 1.75 = 4.311 \ln\left[\frac{0.465 + 0.266}{0.263}\right].$$

$$\underline{\underline{y \simeq 7.51\ m}}$$

which is a good high toss.

Finding the horizontal range of the shuttle-cock,

From (13),

$$y = \frac{V_t^2}{g} \ln\left[\frac{\sin\left[\frac{V_t}{V_{xi}}\left(e^{\frac{gx}{V_t^2}}-1\right) + \tan^{-1}\left(\frac{V_t}{V_{yi}}\right)\right]}{\sin\left[\tan^{-1}\left(\frac{V_t}{V_{yi}}\right)\right]}\right]$$

$$-1.75 = \frac{(6.5)^2}{9.8} \ln\left[\frac{\sin\left[\frac{6.5}{91.36}\left(e^{\frac{9.8x}{6.5^2}}-1\right) + \tan^{-1}\left(\frac{6.5}{23.88}\right)\right]}{\sin\left[\tan^{-1}\left(\frac{V_t}{V_{yi}}\right)\right]}\right]$$

$$-1.75 = 4.31 \ln\left[\frac{\sin\left[0.157\left(e^{0.23x}-1\right) + 0.266\right]}{0.263}\right]$$

$$0.175 = \sin\left[0.157\left(e^{0.23x}-1\right) + 0.266\right]$$

$$\sin(\pi-\theta) = \sin\theta$$

$$\therefore \quad 0.175 = \sin\left[\pi - 0.157\left(e^{0.23x}-1\right) - 0.266\right]$$

$$\sin^{-1}(0.175) = \pi - 0.157\left(e^{0.23x}-1\right) - 0.266$$

$$0.157\left(e^{0.23x}-1\right) = 2.698$$

$$e^{0.23x} = 18.18$$

$$0.23x = 2.9$$

$$\underline{x \sim 12.6\,m}$$

Which is a good range for a long toss stroke.

Therefore, from the results obtained using the equations, we can say that the Taiwanese players are able to hit good clear strokes to the base-line. Similar analysis can be performed on a variety of strokes.

6. Limitations and further work

- The equations can only predict the trajectory and final landing position of the shuttlecock approximately.
- Fast jump smashes do not necessarily follow parabolic trajectory and instead can be described using a straight line.
- Environmental conditions- humidity, temperature, wind direction and air resistance can significantly alter the trajectory.
- Shuttlecock parameters- mass, length, width, geometry and material will influence trajectory.
- Applying research findings on the aerodynamic properties of shuttlecock and badminton strokes can help improve and assist badminton player training and give better prediction of the outcome of the game.

7. Acknowledgements

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9. Supplementary- solved integration

$$(5) \quad \int_{v_{yi}}^{v_y} \frac{dv_y}{\left(1+\frac{bv_y^2}{mg}\right)} = \int_0^t -dt$$

Let $\sqrt{\frac{b}{mg}}\, v_y = \tan\theta \qquad V_T = \sqrt{\frac{mg}{b}} \qquad \therefore \frac{V_y}{V_t} = \tan\theta$

$$dv_y = V_t \sec^2\theta \, d\theta$$

$$\therefore \int_{\tan^{-1}\left(\frac{v_{yi}}{V_t}\right)}^{\tan^{-1}\left(\frac{v_y}{v_t}\right)} \frac{V_t \sec^2\theta \, d\theta}{g\,(1+\tan^2\theta)} = -t$$

$$\tan^{-1}\left(\frac{v_y}{V_t}\right) - \tan^{-1}\left(\frac{V_{yi}}{V_t}\right) = \frac{-gt}{V_t}$$

$$\tan^{-1}\left(\frac{\frac{V_y - v_{yi}}{V_t}}{1 + \frac{V_y v_{yi}}{V_t^2}}\right) = \frac{-gt}{V_t}$$

$$\frac{V_t\,(v_{yi} - v_y)}{V_t^2 + v_y v_{yi}} = \tan\left(\frac{gt}{V_t}\right)$$

$$V_y = \frac{V_{yi} V_t - V_t^2 \tan\left(\frac{gt}{V_t}\right)}{V_t + v_{yi} \tan\left(\frac{gt}{V_t}\right)}$$

$$\boxed{V_y = \frac{V_{yi} - V_t \tan\left(\frac{gt}{V_t}\right)}{1 + \frac{v_{yi}}{V_t}\tan\left(\frac{gt}{V_t}\right)}}$$